Shubnikov-de Haas oscillations of a single layer graphene under dc current bias

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Shubnikov-de Haas (SdH) oscillations under a dc current bias are experimentally studied on a Hall bar sample of single-layer graphene. In dc resistance, the bias current shows the common damping effect on the SdH oscillations and the effect can be well accounted for by an elevated electron temperature that is found to be linearly dependent on the current bias. In differential resistance, a novel phase inversion of the SdH oscillations has been observed with increasing dc bias, namely we observe the oscillation maxima develop into minima and vice versa. Moreover, it is found that the onset bias current, at which a SdH extremum is about to invert, is linearly dependent on the magnetic field of the SdH extrema. These observations are quantitatively explained with the help of a general SdH formula.

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The effect of a dc current bias on the nonlinear response of two-dimensional electron systems (2DES) in a classically strong magnetic field is a subject of current interest. In conventional 2DES, current bias induced effects have been widely studied, in the context of the breakdown of quantum Hall effect, and of some recently discovered nonlinear phenomena such as the Zener-tunneling oscillations, the zero differential states, and the microwave-induced zero-resistance states. Nevertheless, similar studies on 2DES with a single-layer graphene, as shown in Fig. 1(a), imply high homogeneity of the sample.

In our sample, the most observable effects of a dc bias current are on the SdH oscillations. Typical experimental traces are shown in Fig. 2, which were measured at $T = 2.0$ K and with a fixed gate voltage $V_g = -40$ V. As shown in Fig. 2, the magnetoresistance is nearly flat at lower field ($B < 2$ T) and has negligible dependence on dc bias, while the SdH oscillations, occurring at higher fields, are obviously damped by increasing bias current. We show that the damping of the SdH oscillations can be well accounted for by an elevated electron temperature that is found to be linearly dependent on the bias current. In differential resistance, the bias current shows the common damping effect on the SdH oscillations and the effect can be well accounted for by an elevated electron temperature that is found to be linearly dependent on the current bias. In differential resistance, a novel phase inversion of the SdH oscillations has been observed with increasing dc bias, namely we observe the oscillation maxima develop into minima and vice versa. Moreover, it is found that the onset bias current, at which a SdH extremum is about to invert, is linearly dependent on the magnetic field of the SdH extrema. These observations are quantitatively explained with the help of a general SdH formula.

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In the regime of SdH oscillations, the magnetoresistance of a 2DES can be written in a general form regardless of its energy dispersion:

\[ R_{xx} = R_0 \left[ 1 + \lambda \sum_{s=1}^{\infty} D(sX) \exp \left( \frac{s\pi \nu_e T}{\omega_c T} \right) \cos \left( \frac{\hbar S_F}{eB} - s\pi + s\phi_0 \right) \right], \tag{1} \]

where \( \lambda \) is a constant prefactor, \( S_F = \pi k_F^2 \) is the area enclosed by the Fermi circle, \( \omega_c = eB/m^* \) is cyclotron frequency, \( T \) is the lifetime of the carrier, \( s \) denotes the harmonic order, and \( D(sX) \) is the temperature damping factor

\[ D(sX) = \frac{sX}{\sinh(sX)} = \frac{e^{sX}}{\sinh(sX)} \] \tag{2}

In Eq. (1), \( \phi_0 \) accounts for the Berry phase of the 2DES, with \( \phi_0 = 0 \) for conventional 2DES and \( \phi_0 = \pi \) for single-layer graphene. Due to its linear energy dispersion \( \epsilon(k) = \epsilon_F \sqrt{k^2 + (2m^* \hbar^2)/(\pi^2)} \), the effective mass of a single-layer graphene is dependent on the carrier density:

\[ m^* = \frac{\hbar^2}{2V_F^2} \frac{\hbar^2}{eV_F^2} = \frac{\hbar^2}{2V_F^2} \frac{\hbar^2}{eV_F^2}. \tag{3} \]

From Eq. (1), the amplitude of the SdH oscillations at each extremum \( (B_{ex}) \), neglecting higher harmonics to the first order, is given by

\[ A_{ex} = \lambda D(X) \exp \left( -\frac{\pi}{\omega_c T} \right). \tag{4} \]

At sufficiently high temperature such that \( 2\pi^2 k_B T/\hbar \omega_c > 1 \), a linear relation on temperature for the quantity

\[ F(A_{ex}, T) \equiv B_{ex} \ln \frac{\hbar e}{8\pi^2 k_B m^* T} \frac{B_{ex}}{A_{ex}} = -\frac{2\pi^2 k_B m^*}{\hbar e} T + B_{ex} \ln \left( \frac{\lambda m^*}{2m^*} \right) \frac{\pi m^*}{e\tau} \tag{5} \]

follows, which can be used to extract the effective mass \( m^* \), with the theoretic mass \( m^* \) calculated by Eq. (3).

Figure 3(b) shows the plot \( F(A_{ex}, T) \) versus \( T \) for several SdH extrema of the traces presented in Fig. 3(a). It is evident that data from different extrema collapse on the same line at temperature \( T \geq 10 \) K, with a slope corresponding to the calculated effective mass \( m^* = m^* = (\hbar/eV_F)\sqrt{\pi n_F} = 0.0332 m_e \), where the carrier density \( n_F = 3.16 \times 10^{12}/\text{cm}^2 \) is obtained from the measured SdH period and \( v_F = 1.1 \times 10^6 \text{ m/s} \) adopted from literatures.8,13 This excellent agreement testifies the validity of Eq. (1) to describe the SdH oscillations in our graphene sample.

With the effective mass known, Eq. (4) suggests that the lifetime can be extracted from the slope of a log\[A_{ex}/D(X)\] versus \( 1/B_{ex} \) plot. Such plots for the data shown in Fig. 3(a) are
Eq. (3). (c) The of the solid line corresponds to vs. T plot for various SdH extrema as labeled in the graph. The slope reveals that the lifetime is nearly constant in the temperature range.

There is theoretical implication12 that presented in Fig.3(c). The inset of Fig.3(c) indicates that the lifetime, τ, is roughly constant in the studied regime, Eq. (4) gives a formula. In particular, with a constant lifetime in the studied regime of SdH oscillations, as function of bias current. A linear relation with a slope α = 1.07 K/μA is found. The inset shows that, at a given bias current, the electron temperature is roughly constant in the regime of SdH oscillations.

implicating an energy dissipation by the diffusion of the hot electrons into cold electrodes, rather than by the emission of phonons.15 Assuming simply the Wiedemann-Franz law, $k = \rho T$, between the thermal and electrical conductivities, the electron temperature can be qualitatively estimated from the heat balance between the loss by electron diffusion and the joule heating $\nabla (\kappa \nabla T_e) = P_{joule}$.15 And further assuming a quasi-one-dimensional solution along the Hall bar, the electron temperature in the middle of the Hall bar is roughly $T_m \approx R_0/(2\sqrt{\pi}I_{dc}$, where $L = \frac{e^2}{2k_B^2}/(3e^2)$ is the Lorenz number, and $R_0$ is the resistance of the Hall bar at zero magnetic field. Therefore, we can estimate an average electron temperature, $T_e \approx T_m/2 = \alpha I_{dc}$, with

$$\alpha \approx \frac{R_0}{4\sqrt{\pi}} = \frac{\sqrt{3}e}{4\pi k_B} R_0.$$  

Taking the experimental value $R_0 \approx 700 \Omega$, we estimate $\alpha \sim 1.12$ K/μA for the data given in Fig. 2(a), which agrees surprisingly well with the experimental value $\alpha \sim 1.07$ K/μA as obtained in Fig. 4.

Having demonstrated the validity of Eq. (1) for describing the SdH oscillations in the single-layer graphene, and established that the effect of a bias current can be taken into account by an effective electron temperature $T_{e}$, now we are able to focus on the differential resistance that is given by

$$r_{xx} \equiv \frac{\frac{\partial V}{\partial I}}{I_{dc}} = \frac{\partial (R_{xx})}{\partial I} = R_{xx} + I_{dc} \frac{\partial R_{xx}}{\partial I} \tag{8}$$

where $R_{xx}$ is given by Eq. (1) with $T = T_e(I_{dc})$. In the experimental regime, we have found that $R_0(T)$ and $\tau$ are near constant with respect to the temperature or bias current, it follows

$$r_{xx} = R_0[1 + \Lambda \cos(\hbar S_F/e B - \pi + \phi_0)] \tag{9},$$

where higher harmonics of the oscillatory terms have been neglected, and the oscillation amplitude is

$$\Lambda = \frac{\Lambda}{\Lambda} \left[ \frac{D(X_e)}{I_{dc}} + \frac{\partial D(X_e)}{\partial T_e} \right] \exp \left( -\frac{\pi}{\omega_c \tau} \right) \tag{10},$$

with $X_e = 2\pi^2 k_B T_e/\hbar \omega_c$.

The second term in the bracket on the right-hand side of Eq. (10) is proportional to $I_{dc}$ but its sign is negative, opposite...
to the first term, because $\partial D(X_c)/\partial T_e < 0$, and normally we should have $\partial T_e/\partial I_{dc} > 0$. As a result, when the bias current is sufficiently large, the SdH amplitude of the differential resistance can become negative, giving rise to a inversion of oscillation extrema. The onset of the inversion occurs at

$$D(X_c) + I_{dc} \frac{\partial D(X_c)}{\partial T_e} \frac{\partial T_e}{\partial I_{dc}} = 0. \quad (11)$$

In our sample, the electron temperature is linear dependent on bias current, such that the solution of Eq. (11) satisfies $X_e = 1.915$, i.e.,

$$k_B T_e/\hbar \omega_c = 0.097, \quad (12)$$

thus, we have the onset current for phase inversion

$$I_{\text{inv}} \approx T_e/\alpha = \beta B; \quad \beta = 0.097 e/(k_B m^* \alpha), \quad (13)$$

which explains well the observed relation as shown in Fig. 2(b).

Moreover, substitute the observed coefficient $\alpha = 1.07 K/\mu A$ and the effective mass $m^* = 0.0332 m_e$ into Eq. (13), we get a coefficient $\beta = 3.67 \mu A/T$, which reasonably agrees with the value $\beta = 4.2 \mu A/T$ determined from the experimental data.

From the above analysis, we emphasize that the dc-bias-induced inversion of SdH oscillations is unique to the differential resistance measurements, unlike that of magneto-intersubband oscillations where the inversion originates in the dc resistance, as recently discovered in double quantum wells.\(^{17,18}\) It is evident that this phenomenon in differential resistance is generic in 2DES, regardless of their energy dispersion.

We notice that similar dc-bias-induced inversion of SdH oscillations has been observed in conventional 2DES of high mobilities,\(^{19,20}\) where it is believed that the phenomenon cannot be simply described by an elevated electron temperature, rather a nonuniform spectral diffusion has to be taken into account.\(^1\) Our work indicates that, at least for 2DES in the lower mobility regime, the observed phase inversion of SdH oscillations can be well accounted for by an elevated electron temperature.

In summary, we have studied the influence of a dc bias on the magnetoresistance of a single-layer graphene. In dc resistance, electron temperatures extracted from the amplitude of SdH oscillations manifest a linear dependence on the bias current, implicating a dominant heat dissipation mechanism via electron diffusion. In differential resistance, a novel phase inversion of the SdH oscillations has been observed, with an onset bias current that is proportional to the magnetic field.

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\(^{1}\)J. Q. Zhang, S. Vitkalov, and A. A. Bykov, Phys. Rev. B 80, 045310 (2009), and reference therein.


